

Exercise: normal form of the tangent bifurcation

Analyze this dynamical system for its attractors, repellors, and their dependence on the parameter, α :

$$\dot{u} = f(u) = \alpha - u^2$$

1. Make a plot of the dynamics for different values of $\alpha < 0$ and $\alpha > 0$.
2. Determine fixed points, u_0 , as solutions of $\dot{u} = 0$. [You will have to distinguish cases $\alpha > 0$ and $\alpha < 0$.]
3. Determine the stability by examining the sign of $\frac{df}{du}(u_0)$.
4. Make a bifurcation diagram by plotting the fixed points as a function of α and marking them as stable or unstable.